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# THEORY OF THERMAL WAVES

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An analogy between thermal and electromagnetic waves is established. The conditions under which thermal oscillations can give rise to acoustical oscillations are derived. The coefficients of thermal conductivity in liquid helium are calculated as a function of the temperature below the lambda point.

1. Analogy between Thermal and Electromagnetic Waves. We shall examine the first two Maxwell equations in the case that electric and magnetic conduction currents are absent:

$$\operatorname{rot}\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t},\quad \operatorname{rot}\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}. \tag{1}$$

Let the physical properties of the medium be determined by the following equations:

$$\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P},\quad \mathbf{B}=\mu_0\mathbf{H}+\mathbf{M}.$$

Substituting these equations into system (1), we rewrite the latter as follows:

$$\operatorname{rot}\mathbf{H}=\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}+\frac{\partial\mathbf{P}}{\partial t},\quad \operatorname{rot}\mathbf{E}=-\mu_0\frac{\partial\mathbf{H}}{\partial t}-\frac{\partial\mathbf{M}}{\partial t}. \tag{2}$$

In these equations, the second terms on the right determine the electric and magnetic displacement current densities which arise as a result of polarization and magnetization, respectively.

In the stationary case, the system of equations (2) goes over into the following:

$$\operatorname{rot}\mathbf{H}=0,\quad \operatorname{rot}\mathbf{E}=0, \tag{3}$$

which for a two conductor line has a solution of the form [1]:

$$\mathbf{H}=\mathbf{H}_0=H_0(x)\,\mathbf{k},\ \mathbf{E}=\mathbf{E}_0=E_0(x)\,\mathbf{j},\ H_0=E_0. \tag{4}$$

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In the line indicated, the x axis is oriented along the conductor axis, while the y and z axes lie in the transverse cross section plane of the conductor.

Let us assume that the electric and magnetic displacement current densities satisfy a law analogous to Ohm's law: $\frac{\partial \mathbf{P}}{\partial t} = \gamma \mathbf{E}$, $\frac{\partial \mathbf{M}}{\partial t} = \gamma_m \mathbf{H}$ . Then the equations for the nonstationary electromagnetic field assume the form

$$\operatorname{rot} \mathbf{H} = \gamma \mathbf{E} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{rot} \mathbf{E} = -\gamma_m \mathbf{H} - \mu_0 \frac{\partial \mathbf{H}}{\partial t}. \tag{5}$$

We seek the solution of the system of equations indicated in the form

$$\mathbf{H} = I(x, t)\, \mathbf{H}_0(x); \quad \mathbf{E} = \frac{e_1(x, t)}{\varepsilon_0} \mathbf{E}_0(x). \tag{6}$$

It is not difficult to prove the validity of the following equalities:

$$\operatorname{rot} \mathbf{H} = -\frac{\partial I}{\partial x} H_0 \mathbf{j} = -\frac{\partial I}{\partial x} \mathbf{E}_0, \quad \operatorname{rot} \mathbf{E} = \frac{1}{\varepsilon_0} \frac{\partial e_1}{\partial x} E_0 \mathbf{k} = \frac{1}{\varepsilon_0} \frac{\partial e_1}{\partial x} \mathbf{H}_0. \tag{7}$$

Since $E_0$ and $H_0$ do not vanish, substituting (6) into (5) and using (7), we obtain

$$\frac{\partial e_1}{\partial t} + \frac{\gamma}{\varepsilon_0} e_1 = -\frac{\partial I}{\partial x}, \quad \mu_0 \varepsilon_0 \frac{\partial I}{\partial t} + \gamma_m \varepsilon_0 I = -\frac{\partial e_1}{\partial x}. \tag{8}$$

In the particular case when $\gamma = \gamma_m = 0$, the system (8) goes over into the equations first obtained by Mandel'shtam [1].

For a stationary electromagnetic field $e_1 = CU$, and $\epsilon_0 \mu_0 = CL$. Using these expressions, we rewrite the equations in system (8) as follows:

$$L \frac{\partial I}{\partial t} + RI = -\frac{\partial U}{\partial x}, \tag{9}$$

$$C \frac{\partial U}{\partial t} + GU = -\frac{\partial I}{\partial x}. \tag{10}$$

Here $R = \gamma_m \epsilon_0 / C$ and $G = \gamma C / \epsilon_0$.

Equations (9) and (10) determine the voltage drop U and the change in the current strength I on a section $\partial x$ of the two conductor line [2].

Using a well-known procedure, we can obtain the telegraph equation from (9) and (10):

$$LC \frac{\partial^2 U}{\partial t^2} + (GL + RC) \frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} - RGU.$$

Let us rewrite Eq. (9) as follows:

$$I = -\frac{1}{R} \frac{\partial U}{\partial x} - \frac{L}{R} \frac{\partial I}{\partial t}. \tag{11}$$

Now, it expresses Ohm's law for the element dx of the line, in which there is an electromotive self-induction force. Fourier's hypothesis, on the other hand,

$$q = -\lambda \frac{\partial T}{\partial x} \tag{12}$$

corresponds to Ohm's law only in a particular case.

There presently exist several techniques for deriving the heat-conduction equations of hyperbolic type [3-9]. All of these methods contain explicitly or implicitly the hypothesis of thermal relaxation, which in its turn requires the following generalization of Eq. (12):

$$q = -\lambda \frac{\partial T}{\partial x} - \alpha \frac{\partial q}{\partial t}. \tag{13}$$

Comparing Eqs. (13) and (11) establishes the analogy between the thermal flux q and the current strength I and the temperature T and voltage U.

In analogy to (10) the equation for the balance of the quantity of heat must be written as follows:

$$c_v\rho\,\frac{\partial T}{\partial t} + Q(T) = -\frac{\partial q}{\partial x}. \tag{14}$$

The heat source Q(T) must be calculated independently, for example, in terms of the latent heats of mixing reactions, vaporization, melting, and sublimation. These phenomena are equivalent to leakage of charge in a two-conductor line.

In order to obtain the heat-conduction equation, we shall rewrite Eq. (14) using (13):

$$c_v\rho\,\frac{\partial T}{\partial t} = \lambda\,\frac{\partial^2 T}{\partial x^2} + \alpha\,\frac{\partial^2 q}{\partial x\partial t} - Q(T). \tag{15}$$

Let us differentiate Eq. (14) with respect to t:

$$-\frac{\partial^2 q}{\partial t\partial x} = c_v\rho\,\frac{\partial^2 T}{\partial t^2} + \frac{dQ}{dT}\,\frac{\partial T}{\partial t}.$$

The latter relation permits rewriting Eq. (15) as follows:

$$c_v\rho\alpha\,\frac{\partial^2 T}{\partial t^2} + \left(c_v\rho + \alpha\,\frac{dQ}{dT}\right)\frac{\partial T}{\partial t} = \lambda\,\frac{\partial^2 T}{\partial x^2} - Q(T). \tag{16}$$

It should be noted that dQ/dt enters into the coefficient in front of $\partial T/\partial t$ which characterizes the thermal resistance of the material. Since the latent heats can have any sign, it is not difficult to see that the heat source can decrease or increase the temperature oscillations.

Let us introduce the following notation: $p = 1 + \frac{\alpha}{c_v\rho}\,\frac{dQ}{dT}$, $k_0 = k/p$, $k = \lambda/c_v\rho$. Then Eq. (16) can be rewritten as:

$$\frac{\alpha}{p}\,\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = k_0\,\frac{\partial^2 T}{\partial x^2} - \frac{1}{c_v\rho p}\,Q(T). \tag{17}$$

Here the parameter $k_0 = k_0(T)$ should be called the effective coefficient of thermal diffusivity. The more complete equation (17) leads to the following.

If in a thermodynamic system latent heat effects are present, then in determining the coefficient of thermal diffusivity with the help of the parabolic heat-conduction equation, we find that k = k(T). In reality, a wave process, characterized by two constants k and $\alpha$, can proceed in such a system, while the quantity Q(T) must be calculated from other measurements.

Let Q(T) = 0, then instead of (16), we shall have

$$\frac{c_v\rho\alpha}{\lambda}\,\frac{\partial^2 T}{\partial t^2} + \frac{c_v\rho}{\lambda}\,\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}. \tag{18}$$

Comparing (18) with the telegraph equation, we establish the following analogy between the thermophysical and electric quantities: $\lambda = 1/R$, $\alpha = L/R$, $c_v\rho = C$.

2. Thermal Relaxation Time. In the early papers on the use of a hyperbolic type of heat-conduction equation, the velocity of thermal waves was identified with the velocity of acoustic oscillations, while the parameter $\alpha$ was equated to the Maxwellian relaxation time.

However, according to experimental data on propagation of acoustic and thermal waves in liquid helium, it turned out that the velocities of these waves differed by an order of magnitude [10]. It must be admitted that the identification of the thermal relaxation time with the Maxwellian stress relaxation time is not completely justified; thus, the latter is determined for the range of shear velocities, when only energy dissipation mechanisms operate. This is analogous to the experimentally observed fact that the magnitudes of the coefficient of viscosity, found from viscosometric and acoustic measurements, differ strongly. The point here is that according to the methods indicated the viscosity is determined in a different range of shear velocities.

TABLE 1. Thermal Relaxation Time

| Substance | Frequency, $\nu$, MHz | $\alpha\cdot10^8$ sec | $g$, m/sec | $g_0$, m/sec |
|---|---|---|---|---|
| Air | 0,132—0,4 | 0,98—1,69 | 162—132 | 327 |
| Carbon dioxide | 0,304 | 1,93 | 20,1 | 259 |
| Water | 7—250 | 1,27 | 3,32 | 1490 |

For this reason, in order to determine the thermal relaxation time, the hyperbolic heat-conduction equation was used for calculating acoustical dispersion [11]. It is appropriate to recall that in refining Stokes' equation, Kirchhoff in calculating acoustical dispersion closed the equation of heat conduction using Fourier's hypothesis. Discussing this step, he emphasized that in this case, the quantity of heat is determined by the increase in temperature that occurs in the absence of compression. In other words, acoustical action is not manifested in thermal motions of the material medium. On the other hand, closing the heat-conduction equation using hypothesis (13) takes into account the destruction of heat transfer between molecules due to acoustical action.

The following equation was obtained in [11] for the coefficient of absorption of sound waves:

$$\frac{\bar{\alpha}}{\nu^2}=\left(\frac{\bar{\alpha}}{\nu^2}\right)_0+\frac{2\pi^2(\gamma_0-1)(\gamma_0^2-1)}{\gamma_0 g_0}\,\frac{k}{g^2}\,. \tag{19}$$

Here, the additional notation of the Stokes–Kirchhoff equation is introduced:

$$\left(\frac{\bar{\alpha}}{\nu^2}\right)_0=\frac{2\pi^2}{g_0^3}\left(\frac{4}{3}\,\frac{\mu}{\rho}+\frac{(\gamma_0-1)}{\gamma_0}\,k\right). \tag{20}$$

Knowing the difference between the experimental value of $\bar{\alpha}/\nu^2$ and that calculated using (20), it is possible to determine the velocity of the thermal wave g. The numerical values of g permit determining the thermal relaxation parameters α. The results of the calculations indicated are presented in Table 1 for some gases at 0°C and water at 20°C.

<u>3. Conditions for Transmission of Thermal Waves.</u> We seek the solution of Eq. (18) for a damped traveling wave in the form

$$T=e^{-\gamma t}\theta(x,\,t),\quad \gamma=\frac{1}{2\alpha}\,, \tag{21}$$

after which we rewrite Eq. (18) as follows:

$$\frac{\partial^2\theta}{\partial t^2}-\gamma^2\theta=g^2\,\frac{\partial^2\theta}{\partial x^2}\,. \tag{22}$$

If the thermal wave propagates only in one direction, then the function $\theta$ can be represented as

$$\theta=A\cos\omega\left(t-\frac{x}{g_1}\right). \tag{23}$$

Substituting (23) into (22), we determine the velocity of propagation of traveling waves $g_1$:

$$g_1=g\Big/\sqrt{1+\frac{\gamma^2}{\omega^2}}\,. \tag{24}$$

If

$$\frac{\gamma}{\omega}=\frac{1}{2\alpha\omega}\ll 1, \tag{25}$$

then the velocity of propagation of the thermal wave becomes independent of frequency and it must be defined according to the equation

$$g_1=g=\sqrt{\lambda/c_v\rho\alpha}\,. \tag{26}$$

Let us clarify the conditions under which the relation between the amplitudes of oscillations at different frequencies remain unchanged. For this, we call attention to the damping factor $e^{-\gamma t}$, which determines the change in the amplitude of the oscillations. For this purpose, we shall represent the expression $e^{-\gamma t}$ by the function of the path traversed by the wave x, keeping in mind expression (24). Substituting (24) into the exponent of the damping factor $\gamma t$, we obtain

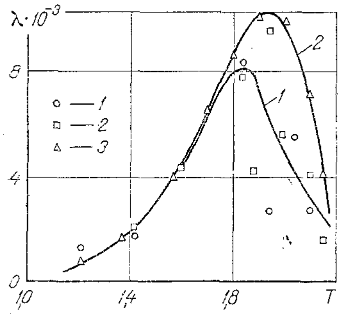

Fig. 1. Thermal conductivity of liquid helium, pressurized by its own saturated vapor: 1) capillary I; 2) capillary II; 3) calculation using Eq. (30).

$$\gamma t = \frac{1}{2\alpha}\,\frac{x}{g_1} = \frac{1}{2}\sqrt{\frac{c_v\rho}{\alpha\lambda}}\,\sqrt{1+\frac{\gamma^2}{\omega^2}}\,x.$$

It is evident from this equation that in order to transmit the thermal wave without distortions, it is necessary that adsorption be independent of frequency. This will be observed when condition (25) is satisfied.

For the substances indicated in Table 1, condition (25) will be satisfied, if the frequency $\nu \geqslant 10^8$ Hz. Since the temperature is related to density and pressure with the help of the equation of state, thermal oscillations will induce acoustic oscillations.

The problem of thermal generation of elastic vibrations in solids was first studied theoretically in [12].

It is shown in what follows that for liquid helium the thermal relaxation time is on the order of $10^{-3}$ sec, which ensures the absence of dispersion with $\nu \geqslant 10^3$ Hz. This does not contradict the experiments performed by V. P. Peshkov on determining the velocity of propagation of thermal waves in liquid helium.

It is interesting to note that the possibility of exciting acoustic oscillations by thermal oscillations and vice versa was discovered experimentally in liquid helium [10].

Let the heat source be $Q(T) = \beta T$, where the coefficient $\beta$ can have any sign. We shall now rewrite Eq. (16) as

$$a_2\frac{\partial^2 T}{\partial t^2} + a_1\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - a_0 T, \tag{27}$$

where $a_0 = \beta/\lambda,\ a_1 = (c_v\rho + \alpha\beta)/\lambda,\ a_2 = c_v\rho\alpha/\lambda.$

We seek the solution for a traveling wave in the form $T = e^{-\gamma t}\theta(x,\ t),\ \ \gamma = (c_v\rho + \alpha\beta)/2\alpha c_v\rho,$ after which instead of (27) we obtain

$$\frac{\partial^2\theta}{\partial t^2} - K^2\theta = g^2\frac{\partial^2\theta}{\partial x^2}\,. \tag{28}$$

Here $K^2 = \gamma^2 - \dfrac{\beta g^2}{\lambda}\,.$

Substituting (23) into (28), we obtain an expression for the velocity of propagation of the wave

$$g_1 = g\left(1 + \frac{\lambda\gamma^2 - \beta g^2}{\lambda\omega^2}\right)^{-1/2}. \tag{29}$$

It is not difficult to show that the condition for the ratios of the amplitudes of the vibrations at different frequencies to remain unchanged has the following form:

$$\frac{K}{\omega} = \frac{c_v\rho - \alpha\beta}{2\alpha\omega c_v\rho} \ll 1.$$

It follows here that the sign and magnitude of the parameter $\beta$ can have a large effect on the conditions for transmission of thermal waves.

TABLE 2. Thermal Conductivity of Liquid Helium

| $T$, K | $\frac{c_v\rho}{\lambda}\cdot 10^5$, sec/cm$^2$ | $\lambda\cdot 10^{-3}$, W/cm·deg, curve 1 in Fig. 1 | $\lambda\cdot 10^{-3}$, W/cm·deg, calc. acc. to (30) | $\alpha\cdot 10^3$, sec |
|---|---|---|---|---|
| 1,21 | 6,93 | 0,698 | 0,798 | 4,12 |
| 1,37 | 5,51 | 1,647 | 1,647 | 4,72 |
| 1,57 | 5,29 | 3,896 | 3,971 | 4,63 |
| 1,70 | 5,47 | 6,094 | 6,483 | 4,44 |
| 1,80 | 5,84 | 7,992 | 8,669 | 4,35 |
| 1,90 | 8,12 | 7,491 | 10,128 | 3,49 |
| 2,0 | 16,3 | 4,694 | 9,869 | 2,24 |
| 2,1 | — | 4,046 | 7,077 | — |
| 2,15 | — | 1,559 | 4,063 | — |

**4. Coefficient of Thermal Conductivity of Liquid Helium.** There is no unified opinion concerning the magnitude of the coefficient of thermal conductivity of liquid helium at temperatures below the λ-point in the scientific literature. Some researchers believe that in the temperature range 1.7-2.0°K, the coefficient of thermal conductivity has a sharp maximum, whose value is more than 800 times greater than the thermal conductivity of copper at room temperature. In a number of papers by Soviet investigators, this point of view is disputed. A detailed discussion of this problem is presented in [10]. The difficulty in establishing a single point of view lies in the fact that the experimental facts indicate a nonlinear relation between the heat flux and the temperature gradient. The latter circumstance invalidates Fourier's hypothesis and makes it impossible to determine the coefficient of thermal conductivity uniquely using existing techniques.

Using the theoretical representation presented above, we shall examine the experimental data obtained by Keesom and Saris [13], who investigated the dependence of the heat flux on the temperature gradient for liquid helium in a capillary in equilibrium with its saturated vapor over the entire range of temperatures from 1.2°K to the λ-point. They published detailed tables of the functional dependence indicated for two capillaries with different diameter and, in addition, the temperature enters as a parameter. Using the tables, the coefficient of thermal conductivity λ is calculated according to the equation

$$\lambda = \left(\frac{q}{dT/dx}\right)_{\frac{dT}{dx}\to 0}.$$

These experimental points are plotted in the λ and T plane (Fig. 1). The smooth curve 1, which smooths the spread in the experimental data, is drawn through the points. The corresponding values of the coefficient λ are shown in Table 2 as a function of temperature. They were used to calculate the complex $c_v\rho/\lambda$ and the data on $c_v$ and $\rho$ are taken from [10]. The values of the complex indicated are on the order of $10^{-5}$ (Table 2). Therefore, the term $\partial T/\partial t$ can be neglected in (18) and it transforms into the equation for propagation of undamped waves. Then Eq. (26) is valid for the velocity of propagation of such waves, while the numerical values of g in the range of temperatures being examined were first measured by Peshkov [10]. The thermal relaxation parameter α was calculated with the help of the experimental data obtained by Peshkov using Eq. (26) (Table 2). It turned out that α is a weak function of temperature.

If it is assumed that in the temperature range examined $\alpha = 4.72\cdot 10^{-3}$ c = const, then we obtain an equation from (26) for determining the coefficient of thermal conductivity:

$$\lambda = 4.72\cdot 10^{-3} c_v \rho g^2. \tag{30}$$

Using experimental data for $c_v$, g, and ρ, we calculate λ from Eq. (30) (curve 1 of Fig. 1; Table 2).

The qualitative behavior of curves 1 and 2 is the same, but a quantitative difference occurs in the range from 1.8 to 2.1°K. A large spread in the experimental data is observed here.

## NOTATION

**H**, magnetic field intensity; **E**, electric field intensity; **D**, electric displacement vector; **B**, magnetic induction vector; $\varepsilon_0$, dielectric constant; $\mu_0$, magnetic permeability; **P**, polarization vector; **M**, magnetization vector; j, k, unit vectors along the y and z axes, respectively; x, y, z, coordinate axes; $e_1$, charge; I, current strength; C, capacitance; U, voltage; L, inductance; t, time; R, resistance of a two-conductor line; G, coefficient of charge leakage; q, heat flux; λ, coefficient of thermal conductivity; T, temperature; α, thermal relaxation time; ρ, density; $c_v$, specific heat capacity at constant volume; g, velocity of the thermal wave; $\gamma_0$, ratio of heat capacities; μ, viscosity; k, thermal diffusivity; $g_0$, Laplace's value of the velocity of sound; $\bar{\alpha}$, coefficient of absorption; ν, frequency; and ω, circular frequency.

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# HETEROGENEOUS VIBRATIONAL RELAXATION UPON FREE MOLECULAR FLOW OF DIATOMIC GAS PAST CONCAVE BODIES

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The method of successive calculation of multiple reflections is used for investigating heterogeneous vibrational relaxation when a free molecular stream of diatomic gas flows past a concave body.

In the investigation of processes of heterogeneous relaxation in gases it is important to distinguish a class of problems in which this is the principal channel of relaxation. It must be taken into account that the importance of heterogeneous relaxation increases with decreasing gas pressure. Since the terrestrial atmosphere at altitudes $\gtrsim 150$ km is characterized by low densities and pressures, it may be expected that in the physicochemical aerodynamics of bodies flying in the atmosphere at such altitudes problems of nonequilibrium kinetics will arise in which the channel of heterogeneous relaxation is the decisive one.

Gashtol'd [1] distinguished a class of aerodynamic problems in which heterogeneous relaxation of translational energy is important, and a method was suggested for calculating the contribution of this process to the aerodynamic characteristics; this method was based on the successive calculation of multiple reflections.

The present work pinpoints a class of problems of the aerodynamics of the upper atmosphere, in which the principal role is played by the process of heterogeneous relaxation of the vibrational energy of gas molecules, and a method of solving them is suggested; this is analogous to the method used in [1].

We examine the external steady-state problem of flow of a free molecular single-component stream of diatomic gas past a concave body. The surface temperature of the body past which the gas flows is taken to be specified.

It is assumed that the mean translational energy of the molecules in a gas stream coming from infinity is much larger than the mean translational energy of molecules in a gas stream reflected with full accommodation and that it is

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